**Tutorial Class 01 - Introduction to Probability, Conditional Probability and Bayes Theorem**

1. a)

Procedure: 3 data packets are sent

Observation: Sequence of successes and failures

Sample Space:

Each packet delivery is an independent event, meaning the probability is unaffected by previous events. This means that , since the probability of some event does not change depending on whether or not some event has already occurred.

For our purposes, this essentially means that the probability of each packet delivery is always the same. Thus,

In general,

where is the number of successful deliveries and is the total number of packets sent.

b)

Procedure: 3 data packets are sent

Observation: The number of successful deliveries

Event Space:

First consider the case where there are deliveries. This is the event .

There are three possible events where there is successful delivery, , , .

Similarly, there are three possible events where there are successful deliveries, , , .

Now, for the general solution to the problem. We need to consider two values here, the total number of packets being sent, , and the number of successful deliveries, .

The powers of the terms and should be easy to understand following the previous problem. The integer by which everything is multiplied is the important part here. Essentially, we are having to find the number of combinations given that there are a certain number of successful deliveries. If there are deliveries in total of which are successful, we are having to find the number of combinations in which we can ‘choose’ positions from the total of positions. Thus, .

c)

Procedure: Packets are sent repeatedly until a successful delivery is made

Observation: Number of attempts

Event Space:

A single attempt has the probability . For two attempts, the probability is . For three attempts, the probability is .

The general solution is

where is the total number of attempts.

d)

Procedure: Send packets repeatedly until there are 3 successful deliveries

Observation: Number of packets sent

Event Space:

The first possibility is simple.

From the second possibility onwards, what we need to keep in mind is that the last delivery has to be a successful one, more specifically, the last delivery has to be the third successful delivery, since that is when the procedure is complete.

If we consider that packets had to be sent, since the last delivery is known to be a successful one, there are successful deliveries in the first packets. Following the same understand as in part b), we are choosing locations from available spots.

Similarly, for packets being sent, we are choosing locations from the first available spots.

Thus, the general solution is

where is the total number of packets sent and is the number of successful deliveries we are looking for.

2.

There are two ways of looking at this problem.

The first way is far nicer. Essentially, we need two pieces of information: how many ways are there to choose 6 firms out of 14 () and how many ways are there of doing this without picking one of the firms from Chittagong (). The probability of anything is just the number of times the chosen outcome occurs divided by the total number of outcomes. Thus,

Using this format will allow us to generalize the solution so we can apply it to other similar problems.

The first method might be a little difficult to understand, but the second method is more straight forward.

Given that is the event that the th firm picked is not from Chittagong,

Every time we choose a firm that is not from Chittagong, the available number of firms not from Chittagong (the numerator) as well as the total number of firms (the denominator) decrease by . Thus,

3.

Let be the event that the th ball picked is red, and be the event that the th ball picked is blue. Let be the total number of red balls, and be the total number of blue balls.

4.

At first site, this may seem like a simple problem of conditional probability. However, upon closer observation, it will become obvious that this requires the Bayes theorem.

Conditional probability applies when the result of one event depends on a prior event. For example, the probability that we will get three consecutive heads depends on whether or not we picked the biased coin. However, this does not apply in the backwards scenario. Whether or not the coin we picked was biased does not depend on whether or not we got three consecutive heads.

There are two scenarios we need to consider, the one where we picked an unbiased coin and got three heads, and the one where we picked a biased coin and got three heads. Thus, the question is, given that one of the two available scenarios where three heads can occur has occurred, what is the probability that the event was the one with the biased coin.

If is the event where a biased coin is picked, is the event where an unbiased coin is picked, is the event that three heads occur, and is the event that three heads do not occur,

5.

This is also a problem that must be solved using Bayes theorem. Here, the event of whether or not the student is stressed occurs first, and the event of the results of the test depend on the first event. However, the scenario we are given puts the test results first, and is thus not conditional probability.

If is the event that the student is stressed, is the event that the student is not stressed, is the event that the results are positive and is the event that the results are negative,

6.

Let be the event that someone has the disease, be the event that the test results are positive and be the event that a rash occurs. Firstly, notice that can only occur for and , since the medicine is only given to people who test positive. Thus,

If this seems a little confusing, drawing a probability tree should help. (In fact, it is best to draw a probability tree for every problem.)

7.

a)

There are two ways for this to happen, firstly, we could pick a blue ball twice, secondly, we could pick a white ball (which then turns blue) and then a blue ball.

b)

No conclusive answer to this question was given in the tutorial class. Here is the solution I liked best:

We know for a fact that a blue ball is selected in the second round. There is no probability about this. It happens for a fact. For this ball to not have been originally blue, we need to have picked a white ball in the first round, painted it blue, and then picked this painted blue ball in the second round from amongst the eight blue balls. The probability of this happening is . Thus, the probability of this not happening is .

If, instead, we are being asked to find the probability that the ball picked in the second round was originally blue, then this is just . Regardless of what else happens, there will always be just balls in the box that were originally blue. This value can never change. I do not see why this would be the meaning behind the question. It is too simple.

c)

The answer provided for this question is as follows:

The probability that some ball has not been picked for the last five rounds is . The probability that a ball is white is . Thus, the answer is .

I do not fully understand this answer, nor do I entirely agree with it.